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MODELING THE SECOND ORDER NON-STATIONARY RANDOM PROCESSES DEVELOPMENT

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ABSTRACT

Annotation. We applied early developed analytic approach for analysis of third order non-stationary stochastic processes (NSPs) to consider the second order ones. A new constitutive differential equation for the NSPs was derived, and its analytic solutions were obtained. Possibility of periodic combination of non- periodic analytic solutions of the equation was shown. Development of a 2-nd order NSP long before its macroscopically fixed beginning was shown and a possible explanation of the corresponding negative probability values for the time period was proposed. Necessity of further fine-tuned experimental investigations is noted.

Keywords: non-stationary stochastic processes, the third and second order stochastic processes, constitutive differential equation, periodic combination of non-periodic analytic solutions, negative probability values before the processes macroscopically fixed beginning

1.0 INTRODUCTION

It is well known that non stationary stochastic processes (NSPs) are characterized by the time moments of its start and finishing, and are the most spreaded ones in nature. Despite the fact there is no universal theoretical model for their description until now. An important confirmation of all NSPs similarity is the numerous universality observations during various real processes development [1-3]. Besides, successful application of the recently proposed analytic model [4-6] to quantitative description of the cold macroplastic deformation of polycrystalline FCC metals and some aspects of the Universe evolution [7] also should be considered as the confirmation.

The main idea that was the basis of the model approach is the well-known statement about “order” of a NSP [4,7]. In the cases of the thermally activated NSPs, such as the macroplastic deformation of polycrystalline metals, heterogeneous solid state phase transformations in crystalline solids etc., the corresponding NSPs are the “third” order ones e.g. from the physical point of wiev – having three development stages or periods: incubation, intensive development and saturation ones. The proposed by the model action for such NSPs is to equal zero the third derivative of the probability for a considered system to be in a final energy state or – for a NSP to be saturated.

On the other hand, there are situations or physical systems which do not have incubation period in their developments. As examples, the following NSPs may be considered here: the magnetization of paramagnets, mono-layer adsorption phenomena, percolation, the processes related to electrons and photons energy level transitions etc. So, such NSPs should be considered as having mainly two stages or periods of their development. Hence, it is natural to expect the analogous approach may be applied in such cases e.g. by equalizing to zero the second derivative of the corresponding probability.

So, the aim of the article is to verify the assumption about possibility to describe a NSP comprising the two development stages by equalization to zero the second derivative of the probability for the NSP development.

2.0 BASICS OF THE APPROACH FORMULATION

As it was shown in [4,7], in general case, the parameter s(t) defining the probability P(s) to find in upper state a two-energy level system consisting of practically uncountable number of subsystems is expressed as:

$$s(t) = [(dE^o/dx)/(dE(t)/dx)]^n \qquad (1)$$

where $dE^o/dx \equiv$ const is maximal energy gradient which has to act on a system for transferring it in an upper energy state or - a NSP to be saturated;

$dE(t)/dx$ - is a continuously changed external energy $E = E(t)$ gradient acting on the whole system and forcing it to change its current (low) energy state or – to force the NSP to develop;

$n \equiv n(t)$ –is a time–dependent variable, related to degrees of freedom for an energy carrying subsystem.

In terms of a NSP development which takes place in the considered system, the parameter $s(t)$ being always positive and determines the probability P(s) for the NSP to be in the saturated state as [4,7]:

$$P(s) = exp[-s(t)] \quad (1)$$

It was shown also that for the third order NSP [8], or in a case of a 3- stage two energy level systems or a thermally activated NSP, we have, according to the approach, the following constitutive nonlinear differential equation:

$$d^3s/dt^3 - 3\cdot(d^2s/dt^2)\cdot(ds/dt) + (ds/dt)^2 = 0 \quad (2)$$

Analytic solving the equation (2) gives two types {1,2} time dependencies of the parameter s(t), for the considered whole system. The corresponding formulas are:

$$s_{1,2}(t) = ln\,\{A_{1,2}/[2 \pm \{(t/t^o_{1,2}) - 1\}^2]\} \qquad (3)$$

where A1,2 and t°1,2 are constants for each revealed the NSP development regimes or time dependencies of the parameter s(t): {1} and {2}.

Besides, it was ascertained that for each of the above NSP development regime: {1} and {2}, there are solutions of the type:

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$$s_{1,2}(t) = \ln \{A_{1,2}/[2 \pm \{([t \pm t^{*}\ \ ]/t^{o}_{1,2}) - 1\}^{2}]\} \qquad (4)$$

where $t^{*}_{1,2}$ is a constant, and in general case: $t_{1,2}^{*} \neq t^{o}_{1,2}$

Further analysis shows [4,8] that there have to be periodic combinations of the above nonperiodic solutions

(4). Such a problem was solved earlier in [4] by assuming for the 2-nd the NSP development regime the periodic realization the solutions of the type (3). Such results are in a good accordance with the commonly accepted data about consecutive participating new grains of a polycrystal in its plastic flow as well as existence of numerous universes excepting our visible one.

3.0 RESEARCH RESULTS AND DISCUSSION

Application of the above approach to describe a NSP development in the two level system without the incubation stage leads to the following results. At first, the constitutive equation describing a "second" order NSP is as follows:

$$d^{2}s/dt^{2} - (ds/dt)^{2} = 0 \qquad (5)$$

The analytic solutions of the equation (5) in general form are as:

$$s^{*}\ \ (t) = \ln \{C\ \ /[(t \pm t^{**})\ /t^{o*}\ \ \ \pm 1]\} \qquad (6)$$

where t**1,2 , t°*1,2 – are constants, and in general case: t**1,2 □ t°*1,2

Graphically, the dependences (6) together with time dependencies of P(t) are shown on Fig.1. So, the revealed by analytical solving of the constitutive equation (5), two types of the time dependencies (6), should be associated with two different regimes of the corresponding NSP development:

$$s^{*}_{1}\ (t) = \ln \{C_{1}\ /[(t \pm\ t^{**}_{1}\ )/t^{o*}\ \ - 1]\},\quad P_{1}\ (t) = \exp[-s^{*}_{1}\ (t)] = \ C\ /[(t \pm t^{**}_{1}\ )/t^{o*}_{1}\ \ - 1]$$
$$s^{*}_{2}\ (t) = \ln \{C_{2}\ /[(t \pm\ t^{**}_{2}\ )/t^{o*}\ \ + 1]\},\quad P_{2}\ (t) = \exp[-s^{*}_{2}\ (t)] = C\ /[(t \pm t^{**}\ )/t^{o*}_{2}\ \ + 1]$$

Further analysis shows also that according to the known procedures [6] one may write:

$$d\, s^{*}(t)\ /dt = L(s, s'), \qquad ds^{*}(t)'/\ dt = Q(s^{*}, s^{*\prime})$$

It is easy to see that there are no typical specific points in the phase space for the autonomous system (5), however the solutions of the system have to be periodic ones, stability of which should be defined by using the following relation;

$$dL/ds^{*} + dQ/ds^{*\prime} = (dL/dt)\cdot s^{*}(t)/\{(dn/dt)\cdot[(dE^{o}/dx) - \ln(dE/dx)] + (d/dt)\cdot(dE/dx)\cdot n(t)/(dE/dx)\} \quad (7)$$

Further, taking into account that $dL/dx = ds^{*\prime}/dt = d^{2}s^{*}/dt^{2}$ and making also rational assumption that $dn/dt > 0$ under increasing external energy gradient $:(dE/dx)(dx/dt) > 0$, we shall have at the conditions $dL/dt > 0$ and $(dE^{o}/dx) > \ln(dE/dx)$:

$$dL/ds + dQ/ds' > 0 \qquad (8)$$

Just given relations (7) and (8) show that under some reasonable conditions the above revealed regimes of two stage NSP may be as stable at: $ds^{*\prime}/dt < 0$ and $(dE^{o}/dx) > \ln(dE/dx)$, as – unstable at: : $ds^{*\prime}/dt > 0$ and $(dE^{o}/dx) > \ln(dE/dx)$.

Evidently, also, that under the reasonable condition $dE^{o}/dx > \ln(dE/dx)$, we have for the most constants values in the cases when $dE^{o}/dx \geq dE/dx$, together with the calculation results ds*□/dt > 0 shown on Fig.1. As it was mentioned above, satisfaction of the both above conditions provide unstable combinations of the constitutive equation solutions of the type (6). Besides, based on the above definition of the parameter s for both the NSP development regimes, we may evaluate a tendency of external force effect on a two-level system behavior or the NSP development, under the conditions of simultaneous increasing the force and the number of degree of freedom for an energy carrying subsystem. Corresponding graphs are shown on Fig.2. As it seen from the graph shown on Fig 2, b at the most investigated values of the model constants, the probability for a NSP to develop is negative in its pre-beginning development stage. In other words, there may be far before (t < 0) the macroscopically fixed beginning a NSP, the process has to develop within a considered system. A possible explanation is that a very weak interaction, possibly – quantum mechanical one, may develop between subsystems transferring a macroscopically measurable amount of an energy in the course of a NSP, and their environment. Evidently, additional investigations are necessary for the results.

4.0 CONCLUSIONS

1. A novel analytic approach for a NSP development description, early successfully applied to the analysis of the macroplastic deformation of polycrystalline metals, was used to describe 2-nd order NSPs in two energy level systems.
2. The new relevant constitutive differential equation for the NSPs was derived, and its analytic solutions were obtained.
3. Possibilities for the solutions to be stable and unstable were shown depending on the model constants values.
4. Development of a 2-nd order NSP long before its macroscopically fixed beginning was shown and a possible explanation of the corresponding negative probability values of the NSPs development for the time period was proposed.
5. Necessity of further experimental research in the field is emphasized.

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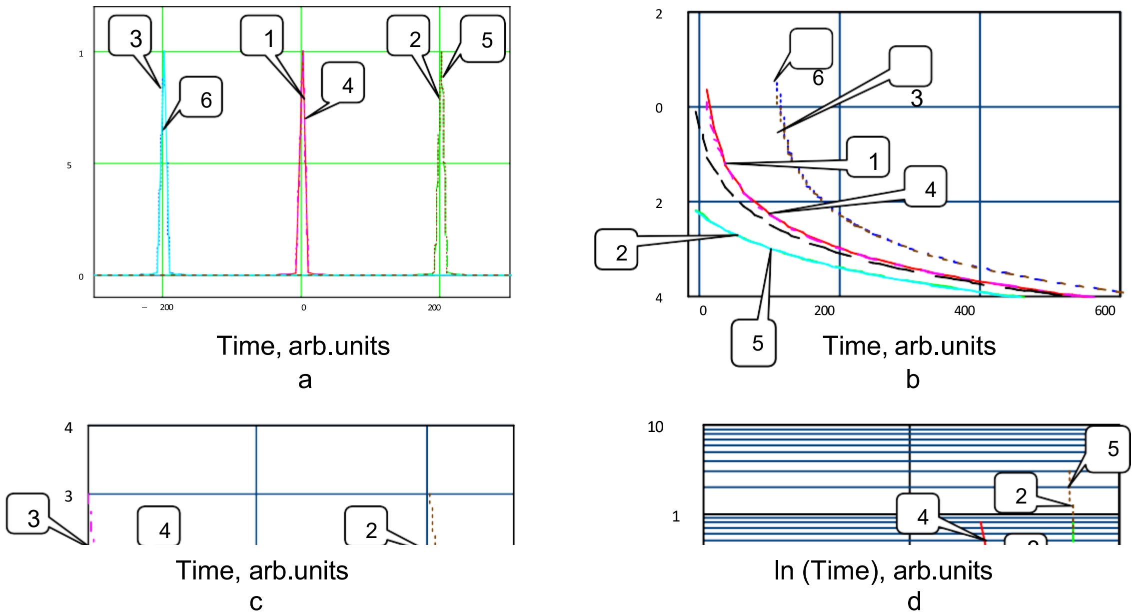

Fig.1. Time dependencies of the calculated values of a) $ds_{1,2}'/dt = d^2s_{1,2}/dt^2$, b) $s_{1,2}(t)$, c) $s_{1,2}(t)$, d) $ln[s_{1,2}]$ under the same values of the corresponding groups (numbers near the curves) of constants: 1)- $t^*_{1,2} = 1$, $tº_{1,2} = 0$, $C_{1,2} = 20$; 2) - $t^*_{1,2} = 1$, $tº_{1,2} = +200$, $C_{1,2} = 20$; 3) - $t^*_{1,2} = 1$, $tº_{1,2} = -200$, $C_{1,2} = 20$; 4) - $t^*_{1,2} = 1$, $tº_{1,2} = 0$, $C_{1,2} = 20$; 5) - $t^*_{1,2} = 1$, $tº_{1,2} = +200$, $C_{1,2} = 20$; 6) - $t^*_{1,2} = 1$, $tº_{1,2} = -200$, $C_{1,2} = 20$

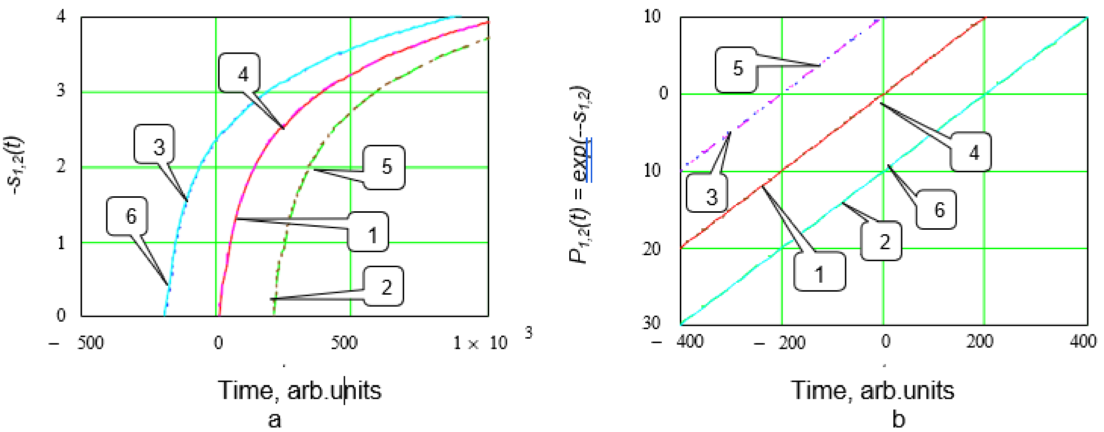

Fig.2. Time dependencies of the calculated values of: a) $s_{1,2}(t)$, b) $P_{1,2}(t) = exp[-s_{1,2}(t)]$ under the same values of the corresponding groups (numbers near the curves) of model constants: 1)- $t^*_1 = 1$, $t°_1 = 0$, $C_1 = 20$; 2) - $t^*_1 = 1$, $t°_1 = +200$, $C_1 = 20$; 3) - $t^*_1 =1$, $t°_{1,2} = -200$, $C_1 = 20$; 4) - $t^*_2 = 1$, $t°_2 = 0$, $C_2 = 20$; 5)- $t^*_2 = 1$, $t°_2 = +200$, $C_2 =20$; 6)- $t^*_2 =1$, $t°_2 = -200$, $C_2 = 20$.